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Short communication

Stable spherically symmetric static charge separated configurations in the atmosphere: Implications on ball lightning and earthquake lights

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ABSTRACT

A theoretical model is presented to show that spherically symmetric and dynamically stable charge separated structures of net zero charge that store energy could be formed by balancing of electrostatic forces and air pressure. The model evaluates the stored energy, the magnitude of separated charge and the pulsation frequency in terms of one parameter, which is a characteristic linear dimension of the system. Implications of the model on ball lightning and earthquake lights are discussed.

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1. Introduction

The possibility of forming stable spherically symmetric charge configurations in the atmosphere has been a subject of many investigations, conducted especially to understand unusual atmospheric phenomena such as ball lightning (BL) and earthquake lights [1–7]. Ball lightning or appearance of faintly luminous spheres floating in air during thunderstorm weather, recorded since time immemorial remains an unresolved elusive phenomenon [1–3,5–11]. Another, unexplained and possibly related effect is the observation of luminous globular structures rising to the atmosphere from the ground, immediately before, during or immediately after an earthquake [12–14]. Here again there are authentic records giving details of these incidents associated with historical and recent earthquakes [11–13]. A strange pulsating spherical light is said to have appeared during the 7th April 2011 aftershock of the March 11th Tohoku earthquake in Japan [15]. Although uncontentious explanations are lacking, the association of earthquakes with electrical activity is a reality which needs understanding. The mechanisms involved in stabilization of thunderstorm and earthquake associated BLs could be similar, even if their routes of formation are very different.

In this note it is shown that potential energy could be stored in spherically symmetric charge distributions of net zero charge formed in the atmosphere by balancing electrostatic forces and external pressure. The virial theorem [9,16] is not violated and the charge distribution and the electric field are free of singularities. Implications of this model on unusual electrical phenomena in the atmosphere are considered.

2. Discussion

Consider a spherically symmetric charge distribution in the atmosphere. If the charge density, the pressure and the electric field at the point (r,θ,ϕ) are $\rho(r)$, $P(r)$ and $E = (r/r)E$ respectively. The condition for equilibrium of the volume element $dV = dS\,dr$ where $dS = r^2\sin\phi\, d\theta\, d\phi$ is,

$$\rho\, dS\, dr\, E - dP ds = 0 \tag{1}$$

giving

$$\frac{dP}{dr} - \rho E = 0 \tag{2}$$

The first term in (1) is the force acting on the element due to the electric field and the second term balances the force originating from pressure difference dP. The charge density and the electric field are connected by the relation,

$$\nabla . E = \frac{\rho}{\varepsilon_0} \tag{3}$$

Eqs. (2) and (3) could be solved for P to obtain,

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$$P(r) = \frac{1}{2}\varepsilon_o E^2(r) + 2\varepsilon_o \int \frac{E^2(r)}{r} dr \quad (4)$$

provided the integral in (4) can be evaluated.

For the purpose of a model and to illustrate a well behaved solution of (4), we choose,

$$E(r) - E_o r \exp(-r/r_o) \quad (5)$$

The electric field (5) is singularity free and vanishes at infinity—a necessary physical requirement. Using (3), the charge distribution corresponding to (5) is derived as,

$$\rho(r) = \varepsilon_o E_o (3 - r/r_o) \exp(-r/r_o) \quad (6)$$

The above charge distribution (6) has following properties, $\rho(r) > 0$, $r < 3r_o$ and, $\rho(r) < 0$, $r > 3r_o$. Thus the positive charge Q confined to a sphere of radius $3r_0$ is,

$$Q = \int_0^{3r_0} 4\pi r^2 \rho(r) dr = 4\pi E_o r_o^3 \exp(-3) \quad (7)$$

The diffused negative charge density extending from $r = 3r_o$ reaches a maximum at $r = 4r_0$ and slowly decays to infinity. The total charge is zero because,

$$\int_0^\infty 4\pi r^2 \rho(r) dr = 0 \quad (8)$$

A plot of $\rho(r)$ vs r and a schematic diagram showing charge distribution is shown in Fig. 1.

When (5) is substituted into (4), the pressure field $P(r)$ evaluates to,

$$P(r) = \frac{\varepsilon_o E_o^2}{2} \exp\left(-\frac{2r}{r_o}\right)\left(r^2 - 2rr_o - r_o^2\right) + P_\infty \quad (9)$$

where P_∞ = pressure at infinity (atmospheric pressure). Setting $P(0) = 0$, we obtain

$$P_\infty = \frac{1}{2}\left(\varepsilon_o E_o^2 r_o^2\right) \quad (10)$$

Using (10), (9) can also be written as,

$$P(r) = P_\infty \left\{1 + \exp\left(-\frac{2r}{r_o}\right)\left[(r/r_o)^2 - 2(r/r_o) - 1\right]\right\} \quad (11)$$

The electrostatic energy U_E and the energy due to pressure U_P are,

$$U_E = \frac{1}{2}\varepsilon_o \int_0^\infty 4\pi r^2 dr = \frac{3}{2}\left(\pi\varepsilon_o E_o^2 r_o^5\right) = 3\pi P_\infty r_o^3 \quad (12)$$

$$U_p = \int_0^\infty [P_\infty - P(r)] 4\pi r^2 dr = \frac{1}{2}\left(\pi\varepsilon_o E_o^2 r_o^5\right) = \pi P_\infty r_o^3 \quad (13)$$

Now consider the virial equation [16] for the system given by,

$$\frac{1}{2}\left(d^2 I/dt^2\right) = 2K + U_E - 3U_P \quad (14)$$

where K = kinetic energy of fluid motion, and I = moment of inertia about the origin. As $U_E = 3U_P$ [Eqs. (8) and (9)], the static equilibrium ($K = 0$) satisfies the virial theorem. When isothermal conditions are assumed so that P/ρ = constant (ρ = density), the value of I for the turns out to be,

$$I = \int_0^\infty [\rho_\infty - \rho(r)] 4\pi r^2 dr = 12\pi\rho_\infty r_o^5 \quad (15)$$

where ρ_∞ = density of air at atmospheric pressure. If $T = K + U_E + U_P$ is the total energy of the system, the virial Eq. (18) can also be written as,

$$\frac{1}{2}\left(d^2 I/dt^2\right) = K + T - 4U_P \quad (16)$$

If r_o is varied to $r_o + \delta r_o$, keeping T constant and neglecting K, we obtain,

$$d^2(\delta r_o)/dt^2 = -\left(2P_\infty/5\rho_\infty r_o^2\right)\delta r_o \quad (17)$$

Thus system executes radial oscillations of about the equilibrium position with a frequency,

$$\nu = \frac{1}{2\pi r_o}\sqrt{\frac{2P_\infty}{5\rho_\infty}} \quad (18)$$

The possibility of oscillations indicates that the equilibrium is stable.

All the characteristics of the system are determined in terms of one parameter r_0, which is a linear dimension. Setting $r_0 = 1$ m, we obtain following values for separated charge $Q = 4\pi(2P_\infty \varepsilon_0)^{1/2}$

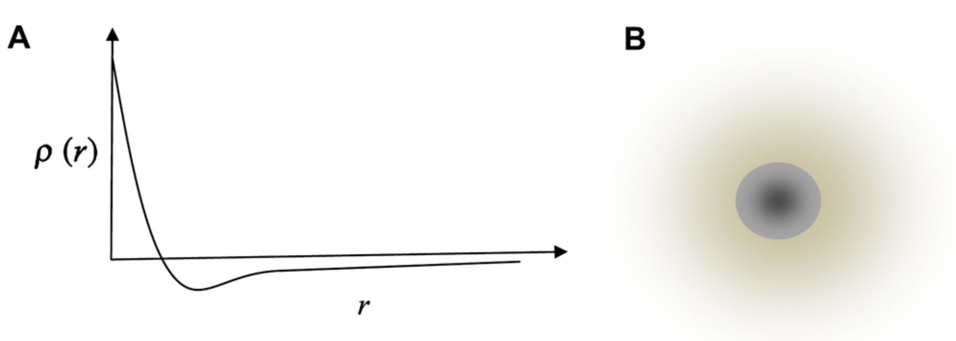

Fig. 1. Plot of $\rho(r)$ vs $r(A)$ and a schematic diagram depicting the spatial charge distribution, the positive charge is confined to the sphere shaded dark gray and the lighter diffused region represents the negative charge (B).

$r_0^2 \text{Exp}(-3) = 8.5 \times 10^{-5}$C, total energy $U = U_E + U_P + 1.26 \times 10^6$ J and $\nu = 14.7$ s^{-1}. Most reports on earthquake BL suggest that these structures have dimensions of order of few meters [13,14], whereas those associated with thunderstorms are smaller [13,5] (diameter 10–15 cm), and therefore according to the model they have energies of the order 10^3 J.

At points in the structure where the electric field exceeds the breakdown field strength of air E_B(=3 × 10^6 Vm^{-1} at ambient air pressure), a corona discharge will take place, naturally accounting for the luminosity of the object. Using (5) and (10) it is seen that the $E(r) > E_B$ is satisfied if when r less than approximately 15 times r_0. The eventual disintegration of the system is due to gradual recombination of separated charges, generating a radial current density $J = \sigma E$. However, this does not happen spontaneously, because decrease of U_E and U_P is counterbalanced by an increase in K to satisfy the virial equation. From (3) and the continuity equation,

$$\nabla \cdot J + \partial\rho/\partial t = 0 \tag{19}$$

it follows that charge density decays as Exp($-t/\tau$) with $\tau = \varepsilon_0/\sigma$. Conductivity of air is ~8 × 10^{-15} Sm^{-1}, giving $\tau \sim 10^3$ s. As conductivity decreases due to ionization process in the corona discharge, the above is only an upper limit for τ and the actual life time could be orders of magnitude smaller.

In the model presented above, we have not taken into account how the charge is associated with matter (gas). The theory governing the condition of equilibrium is based two equations [Eqs. (2) and (3)] constituted of three fields $E(r)$, $\rho(r)$ and $P(r)$. The explicit determination of these quantities is possible, when the coupling of charge to matter is taken into terms of an additional constitutive relation connecting the field variables. In actual situations charge could be associated as ions, electrons, charged dust liquid droplets or combustible particles. The idea presented suggests that dynamically stable charge distributions could be produced in the atmosphere by many mechanisms.

As the net charge is zero, the structure is effectively a capacitor and resembles a thundercloud but spherical in shape. In the above discussion we have neglected the gravitational effects which are important in creation of thunderclouds. Presumably, when the characteristic linear dimension (e.g. r_0) exceeds a critical value, the spherically symmetric structures become unstable. A distribution of spatially separated negative and positive charges, resolves the major problem of plasma models of ball lightning. Normal plasma constituted of ions and electrons cools down almost 'instantaneously' unless there is an external source of heat, therefore cannot explain ball lightning [8]. Again significantly high non-zero net charge also disfavors properties generally attributed to BL like phenomena [7].

It is also interesting to note the analogy of the ball lightning stabilizing mechanism presented above to the hydrodynamic equilibrium of gas clouds in space due to balancing of the opposing forces of gravity and gas pressure. Since the electromagnetic interaction is about 38 orders of magnitude greater than gravity, a stable ball can be produced on much smaller scale than a gas cloud in space.

3. Conclusion

The model presented above shows that spherically symmetric dynamically stable structures could be formed in the atmosphere via balancing of electrostatic forces and air pressure without violation of the virial theorem. Stored energy, magnitude of the separated charge, pulsation frequency is expressed in terms of one parameter, defining a linear dimension of the object. Corona discharge in the space between the separated charges makes the object luminous. Object has a transient existence because of the recombination, but the virial theorem assures that recombination will not lead to spontaneous instability. The purpose of the model is to illustrate the possibility of forming dynamically stable, spherically symmetric charge separated configuration in the atmosphere. A constitutive relation pertaining to the charge-matter system is not incorporated and mention is not made of any specific mechanism by which these structures could be formed. The idea presented in this paper is expected to motivate investigations into nature of BL and related phenomena in the atmosphere.

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