# Ball Lightning as a self-confined light 

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#### Abstract

We demonstrate that the present scientific knowledge is sufficient to derive the conclusion about the existence of the self-confined light in the form of thin spherical layer of strongly compressed air where the intensive white light is circulating in all possible directions and compresses the air. Behaviors of Ball Lightning and the self-confined light in the terrestrial atmosphere are identical.


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## 1. Introduction

Seemingly, the term "self-confined light" in the title is an oxymoron, like dry water or hot ice. Everybody knows that the light travels in straight lines and the cross-section of a light beam increases gradually due to the diffraction divergence. At the same time it is known that there is a phenomenon of self-focusing in a nonlinear optical medium, the refractive index of which increases in the regions where the light is propagating [1]. The phenomenon entails decreasing the cross-section of light beam. In 1971 it was shown theoretically that these two phenomenon can compensate each other in a quadratic nonlinear optical medium where the increase of the refractive index is proportional to the intensity of light [2]. A plane light beam propagating in a homogeneous quadratic nonlinear optical medium along the $z$-axis in the plane $y z$ acquires width $w$ along the $x$-axis. The width depends on the intensity of the beam. You can imagine that the beam produces a planar waveguide in the homogeneous optical medium, the profile of which corresponds to the profile of the beam in the $x$-direction. We can say that the beam confines itself in one direction ( $x$-direction). Thus, there is a fundamental opportunity to obtain the simplest self-confined light.

The possibility of an existence in a nonlinear optical medium conventional white light that confines itself in three dimensions

[^0]is discussed here. In this case the light is located within a sphere of radius about several centimeters. Of particular interest is the nonlinear optical medium in the form of conventional atmosphere air. In this case we can compare properties of the self-confined light with that of Ball Lightning, puzzling and intriguing behavior of whose has not been explained till now. In this case, the selfconfined light can be imagined in the form of a spherical thin layer of strongly compressed air where the intense white light is circulating in all possible directions is possible. The refractive index of the compressed air is greater than that of the surrounding space. The air shows itself as a planar waveguide, curvature of which is different from zero. The layer prevents radiation of the light in free space. On the other hand, the intensive circulating light compresses the air due to the electrostriction pressure. Thus, one can imagine the light that is confined by the trap produced the same light.

The light is radiated gradually in free space due to the phenomenon of molecular light scattering [1]. As a result, the spherical layer is glowing. The light energy stored in the spherical layer decreases gradually. When the energy has become smaller than a certain threshold, the spherical layer becomes instable and disappears traceless. The light radiates in all sides of free space. The compressed air expands sharply. We can wait that properties of the spherical layer where the light plays a decisive role are different from that of conventional material bodies consisting of material particles like molecules, electrons, ions, clusters and so on.

On the other hand, for two centuries scientists are trying to explain a puzzling and intriguing phenomenon of the nature known as Ball Lightning (BL) [3-5]. Unambiguous properties of BL are
similar to those of our spherical layer. First, as the name Ball Lightning suggests, its form is spherical. Second, BL radiates the light also. Third, BL ceases to exist abruptly. At last, BL has anomalous properties, which scientists cannot explain.

Physicists study the phenomenon over two centuries, above 200 various theories are proposed, above 2000 papers and reports are published but simple questions cannot be answered. Why BL may move upwind, how BL penetrates in a room through windowpanes, chimneys, small splits and holes, why BL radiates white light, the spectrum of which corresponds to the temperature of several thousand degrees and is relatively cold at the same time? How does BL catch up a flying airplane and penetrates within its salon? How can BL store extremely great amount of the energy? The list of the questions can be easily continued. We need to agree with the conclusion of Sagan - authors of the last book about Ball Lightning entitled "Ball Lightning: Paradox of Physics" [6]. All theories have one thing in common-none of them work.

There is an understanding that a majority of the theories ought to be withdrawn and efforts ought to be concentrated on hypothesis which satisfy certain requirements.

The first requirement is the ability of BL to penetrate through windowpanes. As is known, any particles such as molecules, electrons, ions, clusters, etc. cannot penetrate through glass. Because of this, either BL does not contain such particles or BL can generate such particles at opposite sides of a windowpane.

The second requirement is the ability of BL to move upwind and to be not blown down by air streams. Any object consisting of material particles is blown down by air streams. On the contrary, BL can accompany a flying airplane. In this case, BL is not blown by the wind, the speed of which surpasses the speed of the greatest hurricane. At present, nobody can imagine the object that satisfies these requirements. It is necessary at first to invent such object before investigating its properties.

## 2. Background for an existence of the self-confined light

In 2002, we put forward a hypothesis [7] where BL is considered as a self-confined light. At first glance, this is perfectly unusual object in the form of a spherical self-confined light bubble or Ball Light. Like any bubble, the Ball Light has a shell. Unlike a conventional soap bubble and the ball light considered in [8], no excess pressure is present within the Ball Light volume. Ball Light is the shell itself. Ball Light shell is a compressed air where an intense light circulates in all possible directions. The refractive index $n$ of the compressed air is greater than that of the surrounding air. In fact, the thin film of the compressed air is a thin-film-planar-lightguide whose curvature is different from zero. In turn, an intense light produces the electrostriction pressure in any optical medium, in particular, in the air where the light propagates. The electrostriction pressure is proportional to the light intensity and tends to near the air molecules close together. Thus, the Ball Light shell is a system of the compressed air and intense light. The compressed air provides a confinement of the intense light and the intense light provides a confinement of the compressed air. In other words, Ball Light shell is a self-confined light in the nonlinear optical medium in a form of conventional air. The air pressures within the volume of Ball Light and outside the Ball Light shell are the same and are equal to the normal atmospheric pressure. Planar light guides like thin Ball Light shell are a basis of up-to-date integrated optics [9]. They confine a light propagating within them from radiation in free space.

It is worthwhile to give additional explanation for one seeming unusual phenomenon, according to which light can circulate in a spherical shell from usual air, the refractive index of which is increased all by $1 \%$ in comparison with surrounding space. The
diameter of the shell can be small enough and make some centimeters only. It seems too much unnatural. From daily experience it is observed that light propagates in a terrestrial atmosphere rectilinearly and an increase in the refractive index by $1 \%$ can change nothing. It is valid for all beams. The shell is transparent for all beams incident on it from surrounding space. Any beam passes through the shell without visible change in direction and intensity. The shell is invisible.

There is only one exception. The spherical shell can limit light waves of whispering gallery type circulating within it. These waves are not radiated in free space because of the phenomenon of total internal reflection at the boundary between the shell and air surrounding it. These waves are not radiated from the shell and thus are also invisible. A circulation of these waves can be found out owing to a phenomenon of molecular light scattering. Scattering waves penetrate through the shell and can be observed. That is why Ball Light seems cold, although its spectrum corresponds to the temperature of thousand degrees.

All physicists are compelled to agree with the statement that an increase in the refractive index by $1 \%$ in that area where light propagates can be sufficient for reliable light confinement. This statement has simple experimental confirmation. Really, in a single mode fiber that is widely used in telecommunication for many years, the refractive index of the core where the light wave propagates surpasses the refractive index of the glass coating by a fraction of a percent. If a round loop of some centimeters radius is formed from such fiber, a light wave in the fiber propagates along the trajectory coinciding with the fiber axis. Light does not leave the loop. On decreasing the loop radius, the loop becomes glowing, as a part of the wave begins to radiate into free space. Thus, there are no doubts about the possibility to confine light in a small volume by a small change of the refractive index.

Very deep doubts are cast upon a principal possibility to form a spherical layer of compressed air and to introduce into the layer white light in a form of whispering gallery wave. Unlike a fiber where two faces can be used for introducing light wave, there are no faces in Ball Light shell. As is known, it is very hard to solve significantly simpler problem concerned with exciting a whispering gallery wave in a conventional glass ball. Nevertheless the problem of production of the spherical layer of compressed air and introducing into it the white light in a form of whispering gallery waves has been solved by the nature and we need to find out this solution.

A Ball Light properties not only coincide with BL properties, but also enables to explain all riddles of BL behavior. On the base of generally accepted physical laws, we have investigated Ball Light behavior in the conventional earth's atmosphere theoretically. It turns out that the behavior coincides with puzzling and intriguing behavior of ball lightning. The following puzzles have been explained: ball lightning behavior near the earth's surface, the reasons of its movement in a horizontal direction at a small distance from the earth's surface [10], possibility of ball lightning motion against the direction of the wind, BL property to accompany airplanes and penetrate in their salons [11], ball lightning penetration in rooms through splits in the walls and through window panes [12]. A phenomenon of self-organization of light that is observed in experiments at attempts to obtain BL in a laboratory has been explained [13]. Besides, it was shown that phenomena responsible for BL existence can take place not only in an initial Ball Light model but also in many other objects. For example, the Ball Light shell can consist of other gases, whose refractive index is greater than that of the air. In this case an increase of the refractive index can be obtained at relatively small pressure in the shell [14]. Such amazing coincidence of Ball Light and BL properties gives the grounds to conclude that Ball Lights exist in the nature and, therefore, Ball Lights are stable. It is required only to find out an explanation of

a)

b)

c)

Fig. 1. Steps of transformation of a fragment of plane soliton into spherical one.
their stability in a frame of generally accepted physical laws and conceptions.

## 3. Self-confinement of the circulating light waves by a spherical film

The ball light can be considered as a generalization of flat incoherent optical spatial soliton propagating in the optical nonlinear medium that is the usual atmospheric air. The ball light can be obtained by transformation of such soliton in two stages. In Fig. 1a, a fragment of a conventional plane incoherent soliton is shown. This is a plane light beam of width $w$ that is parallel to the $x y$ plane and propagates along the $z$ axis in the direction pointed by arrow.

At the first stage, it is possible to replace this plane beam by a set of the plane beams propagating in all possible directions parallel to the same $x y$ plane and having the same total intensity I (Fig. 1b). Such replacement does not change the profile of the beam along its width because the profile of the soliton is determined by the light intensity. The total intensity of the light remains unchanged, as the intensity of a superposition of incoherent beams is equal to the total intensity of these beams. In this case, an influence of light on the nonlinear optical medium in Fig. 1a is the same, as in Fig. 1b.

At the second stage, the curvature of the incoherent spatial soliton in Fig. 1b increases from zero up to some finite size $R^{-1}$ (Fig. 1c). Thus, we obtain a spherical spatial soliton. Unlike an infinite spatial soliton in Fig. 1a, the spherical soliton borrows enclosed area of space, and its diameter is equal $2 R$. Nobody imagined until now the existence of a spherical spatial soliton or, in our terminology, ball lights in nature. Nobody mentioned about the investigation of their properties. If we assume that the BL is the experimental confirmation of the existence of such solitons, our task becomes much easier. We are in a situation where the answer is known in advance, and we only have to look for a solution.

Propagation of waves along the closed trajectory resembles essentially the propagation of the so-called "whispering gallery mode" (WGM) wave through a closed waveguide. The waves of the WGM type are known largely as resonance modes used in optical resonators based on glass microspheres. The WGM wave can be represented as a traveling wave moving along the microsphere equator and reflecting repeatedly from the spherical glass-air boundary. Such waves can be excited even in barrel-shaped segments of an optical fiber as well as in cylindrical waveguides. In the latter case, they are termed the tunneling modes. The idea of the WGM wave in a waveguide can be given by a beam traveling inside a spherical glass cylinder along a spiral with a fixed pitch and reflecting repeatedly from the inner walls of the cylinder.

Let us consider an ability of the WGM waves to form closed waveguides of various type in a nonlinear optical medium. Under the classical self-action of light pulses propagating rectilinearly, the duration of the light action on the medium is limited by the light pulse width. For the WGM waves, the duration of the action increases by the factor of millions for the propagation of light along a closed trajectory. In this case, non-linear effects with great time


Fig. 2. Dependence of the refractive index $n$ on the distance from a ball center.
constant can manifest themselves. The electrostriction effect of the increase of the refractive index in the regions where the light beam propagates is one of the above-mentioned non-linear effects. Although the electrostriction forces are practically inertialess, their reverse action on the light propagation conditions, which is connected with the increase in the refractive index, occurs with a delay. This is connected with the initiation of the elastic deformations in the medium, where the velocity of propagation of the deformations is determined by the speed of sound rather than the speed of light.

Let us assume that the plane space optical soliton shown in Fig. 1a is curved in such a way that the spherical optical soliton in Fig. 1c is formed. In this case, WGW is circulating in the spherical layer. The ability of the WGM wave to form in a uniform quadratic medium a waveguide where the wave rotates is not so evident. To be sure that this is possible, let us first consider a ball made of a linear optical medium inside of which WGM wave is rotating. We will consider the waveguide mode with minimal variations along the radius. In this case, the wave is located near the ball's surfaces. Along the radius, the transverse resonance takes place where the field of the WGM wave coincides with the field of standing wave described by the following equation [15]
$\frac{\partial^{2} u}{\partial \rho^{2}}+\frac{1}{\rho} \frac{\mathrm{~d} u}{\mathrm{~d} \rho}+\left(k^{2}-\frac{a^{2}}{\rho^{2}}\right) u=0$
where, $a=\rho_{0} n / \lambda_{0}, k=\omega n / c$.

## 4. Whispering gallery wave in a linear inhomogeneous optical medium

Let us first consider propagation of WGM wave in a linear optical medium, the refractive index of which changes at $\rho=\rho_{0}$ as is shown in Fig. 2. Let the optical medium be presented by a glass ball of radius $\rho_{0}$, where, $n=1.45$ at $\rho<\rho_{0}$ and $n=1$ at $\rho>\rho_{0}$. With such an abrupt change of the refractive index at the ball side surface the existence of the WGM wave at $\rho_{0} \gg \lambda / n$ is beyond question. However, there is a question that needs to be answered. What is the minimal reflection index jump that supports the existence of the WGM wave inside a ball of radius $\rho_{0}$ ?

Let us introduce the variable $v=u / \rho^{1 / 2}$. Eq. (1) can be expressed as
$\frac{\partial^{2} v}{\partial \rho^{2}}+\chi^{2} v=0$.
Eq. (2) is a wave equation describing propagation of a wave in a waveguide with the propagation constant $\chi$ that depends on $\rho$, and is determined as follows.
$\chi=\sqrt{k^{2}-\frac{a^{2}-1 / 4}{\rho^{2}}}$
Non-trivial solutions to such an equation at $u(0)=0$ and $u(\rho) \ll \varepsilon$ as $\rho \rightarrow \infty$, where $u_{\max } \gg \varepsilon, \varepsilon$ is as small as wished and $u_{\max }$ is a maximal value of $u(\rho)$, are discussed in detail in [16]. The main implications from that discussion are as follows. As high as is


Fig. 3. Dependence of the square of the radial component of the propagation constant (a) and the amplitude of the wave (b) on displacement $\Delta \rho$ relatively radius $r_{0}$.
wished, level of reliability of the confinement of radiation in the ring in the vicinity of $\rho=\rho_{0}$ at sufficiently small $\mathrm{d} \chi / \mathrm{d} \rho$ and sufficiently large jump $k^{2}(\rho)$ at $\rho=\rho_{0}$ can be reached.

As an example, the curves for $\chi^{2}(\rho)$ and $u(\rho)$ for a WGM wave with $\rho_{0}=100 \mathrm{~mm}, \lambda / n=0.5 \mu, \Delta n / n=5 \times 10^{-4}$ are shown in Fig. 3a and $b$, respectively. Note that the wave field in the ball is concentrated in the region of the jump at $\rho_{1}<\rho<\rho_{2}$, where, $\chi^{2}(\rho)>0$. This region is limited at both sides with the regions where, $\chi^{2}<0$, in which the wave cannot propagate but attenuates exponentially. It can be stated that those regions are cut-off waveguides. However, one of the cut-off waveguides $\rho_{2}<\rho<\rho_{3}$ has a finite length. Consequently, it can be called a tunnel. The field at the other end of the tunnel attenuates to the level exceeding zero. The field can excite a wave at the segment $\rho>\rho_{3}$. This wave would carry away part of energy from the presumed resonator, that is, such a resonator has inevitable radiation losses. As mentioned above, in most applications of WGM waves the radiation losses are negligibly small.

## 5. Whispering gallery waves in a nonlinear homogeneous optical medium

Using the same approach to the analysis of the capacity for the self-confinement of radiation in the inhomogeneous linear medium we shall get that for the self-confinement of radiation the following condition of the transverse resonance must be satisfied.
$\int_{\rho 1}^{\rho 2} \chi(\rho) \mathrm{d} \rho=\pi / 2$.
For the return points $\rho_{1}$ and $\rho_{2}$ to exist, it is necessary that the function $\chi(\rho)$ has a maximum. Since the second term in the radicand (3) is a decreasing function of $\rho$, the first term in some region must be an increasing function. Actually, in a quadratic medium $k^{2}=\omega^{2}\left(n_{0}+n_{2} I\right)^{2} / c^{2} \cong k_{0}^{2}\left(1+2 n_{2} I\right)$ and $k^{2}$ increases with $I$ where, $I$ is the intensity of the light. Since $I$ is maximal in the middle of the cross-section, then there is a region where, $I$ is an increasing function of $\rho$. The condition of existence of a maximum can be written as $\frac{k_{0}^{2}}{n_{0}^{2}} \frac{\mathrm{~d} n^{2}}{\mathrm{~d} \rho}>\frac{2\left(a^{2}-\frac{1}{4}\right)}{\rho^{3}}$. For the pronounced WGM waves for which the angle of inclination of the wave vector to the plain of the ball crosssection is close to zero, $a=\rho_{0} n / \lambda_{0}, \mathrm{a}^{2} \gg 1 / 4$ and the above condition can be expressed as
$\frac{\mathrm{d} n}{\mathrm{~d} \rho}>\frac{n}{\rho}$.

c)


b)

d)


g)



Fig. 4. (a) and (b) dependencies of square of the propagation constant $\chi^{2}(\rho)$ and light wave amplitude $u(\rho)$, respectively, on distance $\rho$ in the cylinder where there is a step of the refractive index at $\rho=\rho_{0}$; (c) and (d) the same dependencies as (a) and (b), but the change in the refractive index is caused by the wave whose amplitude is shown in (b); (e) and (f) the same dependencies as (c) and (d), but the change in the refractive index is caused by the wave whose amplitude is shown in (d); (g) and (h) the same dependencies as (e) and ( f ), but the change in the refractive index is caused by the wave which amplitude is shown in (e) and (f); Dependencies (b), (d), (f) and (h) are shown in one graph (i). The following values of the parameters have been used at numerical simulation. $\lambda=0.5 \mu, \rho_{0}=0.1 \mathrm{~m}$, the light wave intensity is such that increment of the refractive index because of electrostriction $\Delta n / n=5 \times 10^{-4}$. The scale of $\Delta \rho$ in the horizontal axes is shown in micrometers, the scale of $u(\rho)$ is shown in arbitrary units, the scale of $\chi^{2}(\rho)$ is $\mu^{-2}$.

Thus, for the existence of self-confinement it is necessary that the increase in $\Delta n / n$ because of the action of light takes place in a fairly small region whose length $\Delta \rho$ would meet the condition $\Delta \rho / \rho<\Delta n / n$. From this, one can conclude that the existence of a maximum does not depend on $\rho$ at the given energy of the light wave. Actually, as $\rho$ increases by $\alpha$ times the light wave intensity decreases by $\alpha$ times and $\Delta n$ decreases by $\alpha$ times a well (at the condition that the cross-section of the waveguide is constant). The condition (5) remains valid in this case.

Let us calculate the intensity of the light wave $u^{2}$ near radius $\rho_{0}$ by means of method of successive approximations [16]. Suppose that at an initial stage distribution of the square of the propagation constant $\chi^{2}(\rho)$ corresponds to the abrupt decrease in the refractive index $\Delta n / n=10^{-3}$ at $\rho=\rho_{0}$ as is shown in Fig. 4a. The amplitude of $u(\rho)$ obtained by means of numerical solution of Eq. (2) with given $\chi^{2}(\rho)$ is shown in Fig. 4b.

The next step of iteration is shown in Fig. 4c and d which display respectively the distributions of the square of the propagation constant $\chi^{2}(\rho)$ and the light wave amplitude for a quadratic optical medium where the change in the refractive index along radius $\rho$ is equal to $n_{2} u^{2}(\rho)$, where, $u(\rho)$ corresponds to the distribution shown in Fig. 4b.

Analogously, in Fig. 4e and f are shown respectively the distributions of the square of the propagation constant $\chi^{2}(\rho)$ and the light wave amplitude for a homogeneous quadratic optical medium where the change in the refractive index along radius $\rho$ is equal to $n_{2} u^{2}(\rho)$, where, $u(\rho)$ corresponds to the distribution shown in Fig. 4d.

In a similar way, Fig. 4 g and h show the distributions of the square of the propagation constant $\chi^{2}(\rho)$ and the light wave
amplitude for a homogeneous quadratic optical medium where the change in the refractive index along radius $\rho$ is equal to $n_{2} u^{2}(\rho)$, where, $u(\rho)$ corresponds to the distribution in Fig. 4f. Besides, in Fig. 4 i are shown the curves on the same scale that were given in Fig. 4b, d, f, and h.

The analysis of the above figures shows that the field distribution along the radius tends to some steady-state. Thus, we have shown numerically that there is a self-consistent solution where the whispering gallery light wave forms a spherical waveguide due to the nonlinear properties of a homogeneous optical medium. The formed profile of the refractive index in the waveguide is such that the intensity of the light wave propagating in the waveguide corresponds to the profile of the intensity that is required to form the same profile of the refractive index.

However, this property does not guarantee the stability of the spherical waveguide. An additional analysis is required.

## 6. Conclusion

In our opinion, the main problem that prevents the recognition of existence of the self-confined light is a prejudice based on common sense that the light cannot confine itself in the conventional air atmosphere. Moreover, the light cannot confine itself in a small volume. We have shown that up-to-date scientific knowledge is sufficient to make sure that the self-confined light is possible in principle. Moreover, even terms for designation of this phenomenon are known. But the most convincing evidence of an
availability of the self-confined light is its existence in the nature in a form of Ball Lightning. This assertion is based on the surprising coincidence of behaviors of our imagined self-confined light derived from physical laws and natural Ball Lightnings derived from numerous evidence of eyewitnesses.

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