Physics Letters A 375 (2011) 3537–3540

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

A proposed experiment on ball lightning model

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ARTICLE INFO

Article history:
Received 30 November 2010
Received in revised form 26 July 2011
Accepted 3 August 2011
Available online 10 August 2011
Communicated by A.R. Bishop

Keywords:
Ball lightning
Total internal reflection
Laser field

ABSTRACT

We propose an experiment for strong light amplification at multiple total reflections from active gaseous media.

1. Introduction

Light reflection from an interface between two media is determined by the wave equation and the boundary conditions, which follow from Maxwell's equations. The wave equations for electric, $\boldsymbol{E}$ and magnetic, $\boldsymbol{H}$, fields in a homogeneous medium with constant μ and ϵ are

$$\Delta\boldsymbol{E}(\boldsymbol{r},t)=\frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{E}(\boldsymbol{r},t),\qquad \Delta\boldsymbol{H}(\boldsymbol{r},t)=\frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{H}(\boldsymbol{r},t). \tag{1}$$

Both equations have plane wave solutions

$$\boldsymbol{E}(\boldsymbol{r},t)=\boldsymbol{E}\exp(i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t),\qquad \boldsymbol{H}(\boldsymbol{r},t)=\boldsymbol{H}\exp(i\boldsymbol{k}\cdot\boldsymbol{r}-i\omega t), \tag{2}$$

where $k^2=\epsilon\mu k_0^2$, $k_0=\omega/c$ and c is the speed of light in vacuum. The fields $\boldsymbol{E}$ and $\boldsymbol{H}$ are not independent. They are related to each other by equations

$$\boldsymbol{H}=\frac{c}{\mu\omega}[\boldsymbol{k}\times\boldsymbol{E}],\qquad \boldsymbol{E}=-\frac{c}{\epsilon\omega}[\boldsymbol{k}\times\boldsymbol{H}], \tag{3}$$

and, if $|\boldsymbol{E}|=1$, the length of $\boldsymbol{H}$ is $|\boldsymbol{H}|=\sqrt{\varepsilon/\mu}=1/Z$, where $Z=\sqrt{\mu/\varepsilon}$ is called the medium impedance.

If space consists of two halves with different $\epsilon_{1,2}$ and $\mu_{1,2}$, the wave equations in them (1) are different, and their solutions should be matched at the interface. The matching conditions follow from the Maxwell equations. They require continuity of the components $\boldsymbol{E}_\parallel(\boldsymbol{r},t)$, $\boldsymbol{H}_\parallel(\boldsymbol{r},t)$ parallel to the interface, and $\epsilon(\boldsymbol{n}\cdot\boldsymbol{E}(\boldsymbol{r},t))$, $\mu(\boldsymbol{n}\cdot\boldsymbol{H}(\boldsymbol{r},t))$, perpendicular to it, where $\boldsymbol{n}$ is a unit normal vector. The wave function in presence of the interface is

$$\begin{aligned}\boldsymbol{\psi}(\boldsymbol{r},t)=\Theta(z<0)\big(&e^{i\boldsymbol{k}_1\cdot\boldsymbol{r}-i\omega t}\boldsymbol{\psi}_1+e^{i\boldsymbol{k}_r\cdot\boldsymbol{r}-i\omega t}\boldsymbol{\psi}_r\rho\big)\\ &+\Theta(z>0)e^{i\boldsymbol{k}_2\cdot\boldsymbol{r}-i\omega t}\boldsymbol{\psi}_2\tau,\end{aligned} \tag{4}$$

where the term $\exp(i\boldsymbol{k}_1\cdot\boldsymbol{r}-i\omega t)\boldsymbol{\psi}_1$ with the wave vector $\boldsymbol{k}_1$ describes the plane wave incident on the interface from medium 1, factors $\boldsymbol{\psi}_i=\boldsymbol{E}_i+\boldsymbol{H}_i$ $(i=1,r,2)$ denote sum of electric and magnetic polarization vectors, $\boldsymbol{k}_{r,2}$ are wave vectors of the reflected and transmitted waves, ρ, τ are the reflection and transmission amplitudes respectively, and $\Theta(z)$ is the step function, which is equal to unity when inequality in its argument is satisfied, and to zero otherwise.

The wave vectors $\boldsymbol{k}_{r,2}$ are completely determined by $\boldsymbol{k}_1$. They are determined uniquely by the constants ϵ_i, μ_i, and by the fact that $k_0=\omega/c$ and the part $\boldsymbol{k}_\parallel$ of the wave vectors parallel the interface must be identical for all the waves. In the following we assume that the medium 1 is lossless, i.e. $\epsilon_1\mu_1$ is real, therefore all the components of $\boldsymbol{k}_1$ are also real.

The normal component $k_{2\perp}$ of the refracted wave is

$$k_{2\perp}=\sqrt{\epsilon_2\mu_2k_0^2-\boldsymbol{k}_\parallel^2}=\sqrt{k_{1\perp}^2-(\epsilon_1\mu_1-\epsilon_2\mu_2)k_0^2}, \tag{5}$$

or it can be represented as

$$k_{2\perp}=\sqrt{\epsilon k_1^2-\boldsymbol{k}_\parallel^2}=\sqrt{n^2k_1^2-\boldsymbol{k}_\parallel^2}, \tag{6}$$

where $n=\sqrt{\epsilon}$ is the refractive index, and we introduced relative permittivity $\epsilon=\epsilon_2\mu_2/\epsilon_1\mu_1$.

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2011.08.009

The amplitudes ρ and τ are well known from textbooks (see [1], for example). They are calculated differently for TE-wave, when the incident electric field is polarized perpendicularly to the plane of incidence, i.e. parallel to the interface (it is usually denoted as $\boldsymbol{E}_s$), and for TH-field, when the incident electric field is polarized inside the plane of incidence (it is usually denoted as $\boldsymbol{E}_p$). For both of these cases we have well-known Fresnel formulas

$$\rho_s = \frac{\mu_2 k_{1\perp} - \mu_1 k_{2\perp}}{\mu_2 k_{1\perp} + \mu_1 k_{2\perp}}, \qquad \rho_p = \frac{\epsilon_2 k_{1\perp} - \epsilon_1 k_{2\perp}}{\epsilon_2 k_{1\perp} + \epsilon_1 k_{2\perp}}, \tag{7}$$

and $\tau_{s,p} = 1 + \rho_{s,p}$. In the following we for simplicity assume that $\mu_2 = \mu_1 = 1$, so $\mu_{1,2}$ are excluded from all our formulas.

From (6) it follows that for lossless media when $0 < \epsilon < 1$ is real, the incident wave, for which $\boldsymbol{k}_\parallel$ is within $nk_1 \leqslant |\boldsymbol{k}_\parallel| \leqslant k_1$, is totally reflected from the interface. This happens because

$$k_{2\perp} = iK''_{2\perp} \equiv i\sqrt{k_\parallel^2 - \epsilon k_1^2}, \tag{8}$$

thus the factor $\exp(ik_{2\perp}z) = \exp(-K''_{2\perp}z)$ of the wave $\exp(i\boldsymbol{k}_2\boldsymbol{r})$ exponentially decays, i.e. the refracted wave becomes an evanescent one. Therefore, the energy does not flow inside the medium 2, and due to the energy conservation it must be totally reflected into medium 1.

If the medium 2 is lossy or gainy, the constant ϵ is a complex quantity $\epsilon = \epsilon' \pm i\epsilon''$, with positive ϵ' and ϵ''. In this case outside the total internal reflection (TIR) region ($|\boldsymbol{k}_\parallel|^2 \ll \epsilon' k_1^2$) we have $k_{2\perp} = k'_{2\perp} \pm ik''_{2\perp}$, where for small ϵ'' ($\epsilon'' k_1^2 \ll \epsilon' k_1^2 - |\boldsymbol{k}_\parallel|^2$)

$$k'_{2\perp} \approx \sqrt{\epsilon' k_1^2 - |\boldsymbol{k}_\parallel|^2}, \qquad k''_{2\perp} \approx \epsilon'' \frac{k_1^2}{2k'_{2\perp}}. \tag{9}$$

In the TIR regime, $k'_{2\perp}$ in Eq. (9) transforms into $iK''_{2\perp}$, where $K''_{2\perp} \approx \sqrt{|\boldsymbol{k}_\parallel|^2 - \epsilon' k_1^2}$, and $k''_{2\perp}$ transforms to

$$k''_{2\perp} \to -iK'_{2\perp} = \epsilon'' \frac{k_1^2}{2iK''_{2\perp}}. \tag{10}$$

Therefore, at TIR $k_{2\perp} = \pm K'_{2\perp} + iK''_{2\perp}$, where

$$K'_{2\perp} = \epsilon'' \frac{k_1^2}{2K''_{2\perp}}, \qquad K''_{2\perp} \approx \sqrt{|\boldsymbol{k}_\parallel|^2 - \epsilon' k_1^2}. \tag{11}$$

The '+' sign before imaginary part $iK''_{2\perp}$ determines exponential decay of the refracted wave away from the interface for both lossy and gainy media cases. However the real part, $K'_{2\perp}$ has opposite signs for lossy and gainy cases.

The reflection amplitudes (7) at TIR look

$$\rho_s = \frac{k_{1\perp} - iK''_{2\perp} \mp K'_{2\perp}}{k_{1\perp} + iK''_{2\perp} \pm K'_{2\perp}}, \tag{12}$$

$$\rho_p = \frac{\epsilon_2 k_{1\perp} - \epsilon_1 (iK''_{2\perp} \pm K'_{2\perp})}{\epsilon_2 k_{1\perp} + \epsilon_1 (iK''_{2\perp} \pm K'_{2\perp})}. \tag{13}$$

The positive value of $K'_{2\perp}$ for lossy medium means that the reflection coefficient in TIR is less than one, because part of the energy flux proportional to $K'_{2\perp}$ enters the medium 2 and is absorbed there. The negative value of $K'_{2\perp}$ for gainy medium means that the reflection coefficient in TIR is larger than one, because part of the energy flux proportional to $K'_{2\perp}$, exits the medium 2 and adds to the TIR wave.

In [2] it was incorrectly claimed (see, for example, critics in [3]) that in the case of TIR from a gainy medium the wave vector inside the gainy medium has opposite sign: $k_{2\perp} = K'_{2\perp} - iK''_{2\perp}$, and the reflection coefficient at TIR $|\rho_{s,p}|^2$ is less than unity. If it were so, then the wave function inside the gainy medium would increase proportionally to $\exp(K''_{2\perp}z)$ independently of how small is the gain. Since $K''_{2\perp} \sim 1/\lambda$ (see (11)) then for $\lambda \sim 1000$ nm the intensity of the field inside the gainy medium at a distance 1 mm from the interface would be larger than intensity of the incident wave of light by the factor $e^{2000} \approx 10^{860}$, which surpasses all the astronomical numbers. It proves that the claim in [2] cannot be true.

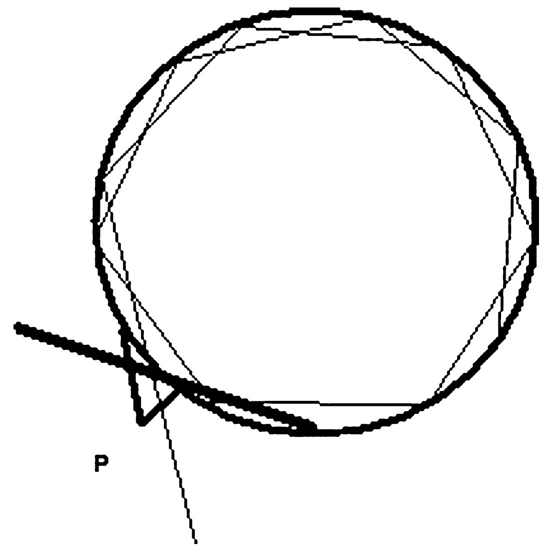

Fig. 1. Schematic of the experiment for multiple TIR off gainy medium.

With the correct sign $k_{2\perp} = -K'_{2\perp} + iK''_{2\perp}$ the reflection coefficient at TIR from a gainy medium is larger than one, and it increases with the gain. The photons induced by the incident wave cannot propagate inside the gainy medium by the same reason as the incident one. They can only go by the tunnelling transmission into the first medium. The increase in the reflected flux is due to the sub-barrier induction of the photon, which tunnels from the gainy medium into medium 1 and coherently adds to the reflected primary photon. The larger is the gain, the larger is the probability of such a process.

2. The proposed experiment for strong enhancement of the light trapped in a glass sphere

The increase of the reflection coefficient at TIR from a gainy medium can be used to develop a curious experiment for storage and amplification of light. Imagine a glass sphere with a coupler P, as shown in Fig. 1. The sphere has thin walls (it is also possible to use a homogeneous glass sphere) and is surrounded by an excited gas (or other active media). The ray of light, shown by the thin line, enters the glass walls through the coupler and then undergoes TIR multiple times. At every reflection the light is amplified according to the analysis in the previous section. It is possible to imagine a geometry in which the ray after entering the glass becomes trapped in it, or after sufficiently many reflections escapes the sphere, as shown by the thick line. The amplification depends on the number of the reflections and on the gain coefficient of the active medium. The number of the reflections is very sensitive to the angle of the incident ray. It is important to note, that the energy accumulates inside the sphere and does not go out of it. The total reflection works like a pump, and the pumped energy density can be much larger than the energy density in the surrounding gainy medium. If the overall amplification is sufficiently high, the glass will melt into a liquid bubble with thin skin filled with the light, similar to the ball lightning described in [4].

We can estimate the magnitude of the light enhancement in such a sphere. Assume that for the active medium $\epsilon_2 \approx 1 - i\alpha$, and

ϵ_1 of the glass is real. For TE-mode, the reflection amplitude at TIR according to Eqs. (12) and (11) can be written as

$$\rho_s \approx \frac{k_{1\perp} - iK_{2\perp} + \alpha k_0^2/2K_{2\perp}}{k_{1\perp} + iK_{2\perp} - \alpha k_0^2/2K_{2\perp}} \approx \frac{k_{1\perp} - iK_{2\perp}}{k_{1\perp} + iK_{2\perp}}\left(1 + \frac{\alpha}{\epsilon_1 - 1}\frac{k_{1\perp}}{K_{2\perp}}\right), \tag{14}$$

where $K_{2\perp} = \sqrt{(\epsilon_1 - 1)k_0^2 - k_{1\perp}^2}$, and the approximation is valid for small α. Similarly we can write equation for ρ_p. For estimating purposes we can assume that in both cases the light is amplified approximately by the factor $1 + 2\alpha$ at each reflection.

Let's consider a sphere of radius $R = 10$ cm submerged into an active medium with small α. The energy I inside it increases with number N of collisions with the walls $\propto (1+2\alpha)^N \approx \exp(2N\alpha)$. The N can be represented via time t as t/t_1, where $t_1 = 2R\sin\theta\sqrt{\epsilon_1}/c$ is the time between two consecutive collisions of photons with the walls, and θ is the grazing angle with surface at the collision point. Therefore $I(t) = I_0\exp(t/\tau)$, where $\tau = t_1/2\alpha = R\sin\theta\sqrt{\epsilon_1}/c$, and I_0 is initial energy of the incident ray. For $\theta = 0.1$ and $\alpha = 10^{-7}$ we get $1/\tau \approx 3\times 10^3$ s^{-1}. Therefore, for $I_0 = 10^{-19}$ J, the energy I after 20 ms reaches 10 MJ.

The following analysis is used to estimate α. The amplification of a laser beam along a path l inside a gainy media is $\exp(2k''l)$, where k'' is the imaginary part of the wave number, and $g = 2k''$ is called the gain coefficient. In a medium with $\epsilon = 1 - i\alpha$, the gain coefficient is $g \approx \alpha k = 2\pi\alpha/\lambda$, where λ is the wavelength. For N_2, CO_2 gas lasers, the gain coefficient is approximately 10^{-2} cm^{-1} [5]. For $\lambda/2\pi \simeq 10^{-4}$ cm we obtain $\alpha = 10^{-6}$.

In the past, many experiments were performed with the whispering gallery mode resonators (WGMR) of small dimensions (~ 1 mm) and large Q-factors (up to $Q \sim 10^{10}$) [6–10], where light undergoes large number $N \sim Q$ total internal reflections. In a larger sphere submerged into an active medium with $\alpha \sim 10^{-7}$, the Q-factor can also be large if the losses of the light ray between two collisions with the interface are less than the gain at a single collision. In such a case we can accumulate enormous energy. The stored photons will heat and melt the resonator, but the electrostriction forces will hold the melted substance together. One can expect to see many interesting nonlinear phenomena in such systems, which are very important both for fundamental and technological physics.

We did not consider application of spherical harmonics for description of the light ray enhancements, because the radius of sphere is much larger than the wavelength, and because the glass shell can have a form different from an ideal sphere. More over, application of spherical harmonics presupposes that distribution of the field is periodic, i.e. the path of the light ray is closed, whereas in the nonstationary case it not necessary.

We considered a special port or a spot on the glass to insert the light in whispering gallery mode, but it is also possible to insert the light with the help of frustrated total reflection.

3. Conclusion

We considered total internal reflection (TIR) of light inside a glass sphere submerged into an active medium. We have shown opposite to assertion of [2] that at TIR the reflection coefficient of the light resonant to excitations of the surrounding active medium is larger than unity. Therefore the light inside the glass sphere in whispering gallery mode (WGM), pumps energy from the active medium. Accumulated energy can reach very high values. It happens, when losses of light in glass on flight path between two collisions with walls are less than gain at a single collision. For estimation of losses we can use coefficient of attenuation of light in transparent media known in literature. In [11], for instance, it is shown that in optical fiber attenuation of an infrared light with wavelength 1310 nm can be of the order 0.4 dB/km [11]. If exponential decrease of the light is described by $\exp(-qL)$, then 0.4 dB/km means that $q \approx 10^{-6}$ cm^{-1}. Therefore the losses in a sphere of radius 1 cm on the chord $l = 2r\sin\theta = 2$ mm for $\theta = 0.1$ is of the order $2\cdot 10^{-7}$ and the gain $\alpha = 10^{-6}$ in (14) is sufficient to overcome the losses. Of course, if the matter of glass is worse, the density of excitation energy in surrounding gas must be higher, or it is possible to have a profit from the fact that the closer is θ to critical angle the higher becomes the amplification.

The estimated total built up energy looks somewhat unrealistic. We do not insist that it can reach precisely such a value 10 MJ, however even if it will reach the value of the order 10 kJ, the experiment will be interesting. First, it will help to check the model of the ball lightning [4], which is a thin spherical compressed gas shell filled with photons in a state of one or several WGMs. If density of photons is high, then photons create surface tension, which keeps the shell in an integral state. The surface tension arises because of electrostriction forces created by attractive interaction $U = -\boldsymbol{dE} = -\alpha E^2$ between electric field and atoms, where $\boldsymbol{d} = \alpha\boldsymbol{E}$ is induced electric dipole moment in an atom, and α is its polarizability. The matter shell filled with high density photons in a coherent state has a long life time, because, if a photon excites an atom, the excitation, due to laws of electrodynamics, immediately emits the photon into the most populated mode. For stability of the considered model of the ball lightning it is not necessary to have an active medium or plasma around it. However, if environment contains excited atoms, resonant to photons in the shell, these atoms increase stability of the shell, the ball lightning moves toward such atoms and feeds itself. All such wonderful properties of the ball lightning model were considered in [4].

It is not necessary that in the proposed experiment one immediately obtains a stable liquid spherical shell filled with the photons, however in any case it is interesting to look how much energy it is possible to accumulate, what technological problems must be solved to increase accumulation, how Q-factor depends on the form of the matter shell, and what nonlinear effects become observable. Usually it is believed that only microspheres can have high quality factor. The surface of macrospheres cannot be ideal and the photons will go out of TIR regime, however the time of staying in WGM mode can be high, because the range of TIR angles is wide and, more over, the presence of outside active medium all the time feeds those photons which remain in the WGM mode. So though the outcomes of the proposed experiment cannot be predicted precisely, all the arising technological problems, we believe, can be solved, and the results of the experiment will not only satisfy the scientific curiosity, but also can be very important for applications. For instance, we can hope to get there a new kind of an engine. Relevance of the considered effect for science and technology seems to be of the same level as superconductivity.

Finally, we were talking about pumping of energy in a sphere, but it is also possible to consider another forms of glass, for instance a toroid. The last form may happen even better than the sphere.

Acknowledgements

We are grateful to Prof. Guenakh Mitselmakher of Florida University for his interest and support, to Prof. Lukas Novotny, Dr. Svetlana Lukishova of Rochester University and to Prof. Vladimir Kadyshevskiy from JINR for their interest, advices and assistance.

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